

COSMOLOGICAL PARAMETERS AND THE WMAP DATA

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I discuss whether the standard cosmological models fit the WMAP data well enough to justify parameter estimation with standard assumptions. The observed quadrupole is low (but has significant foreground uncertainty) and drives weak evidence for theoretical models predicting low values, such as models with a running spectral index. Other more seriously outlying points of the WMAP power spectrum appear not to fit the expectations of simple Gaussian models very well. The effective temperature chi-squared is however acceptable on large scales. There also appears to be evidence for an anisotropic distribution of power, which taken together with the other points may indicate that either there is a problem with the WMAP data or that standard cosmological models are incorrect. These issues should be clarified before cosmological parameter extraction for the usual standard models can be trusted, and hint that maybe the CMB is more interesting than we imagined. I also discuss various systematic and analysis issues, and comment on various oddities in the publicly available first year WMAP data and code.

1. Introduction

The recent results of the Wilkinson Microwave Anisotropy Probe (WMAP)¹ provide full-sky maps of the CMB anisotropy together with various foregrounds and noise. Taken at face value the anisotropy power spectrum can be used to tightly constrain various combinations of cosmological parameters. Partial parameter degeneracies can be broken by including additional data from other sources, giving good constraints on many parameters individually^{2,3}. However data from Lyman- α forest is currently plagued by systematic issues (for example see Ref. 4), so joint constraints including only statistical errors should not be taken too seriously. Even cosmic shear results, which in principle are a clean probe of the total matter distribution and should give robust joint parameter constraints³, are currently subject to various observational systematics and uncertainties that can be hard to quantify. Constraints on the matter power spectrum from 2dF⁵ are sensitive to how the bias is modelled, with conservative results that are marginalized over the bias⁶ giving significantly higher results for the matter density than when more complicated modelling is applied⁷. Having said this, there is a remarkable agreement between the cosmological parameters inferred from many of these data sets and different CMB power spectrum measurements, indicating that the basic flat Λ CDM model with

approximately power-law isotropic adiabatic Gaussian primordial fluctuations may be on the right track.

In principle the CMB anisotropy measurement on large scales should be the cleanest and most direct probe of primordial physics. Usually the sky maps are compressed into power spectra for the purpose of likelihood evaluation, which should be nearly optimal if the fluctuations are indeed isotropic and Gaussian, and the power spectrum and likelihoods are evaluated consistently. Given a function for computing the likelihood from a theoretical power spectrum, computing the posterior parameter constraints is straightforward using the publicly available CosmoMC code^{8a}. However as discussed by Ref. 2 and other authors the WMAP power spectrum has some unexpected features, and seems to have a rather high χ^2_{eff} fit to standard models. I discuss these features in more detail below, where bulleted points are of a technical or trivial nature and may be skipped.

2. Temperature power spectrum

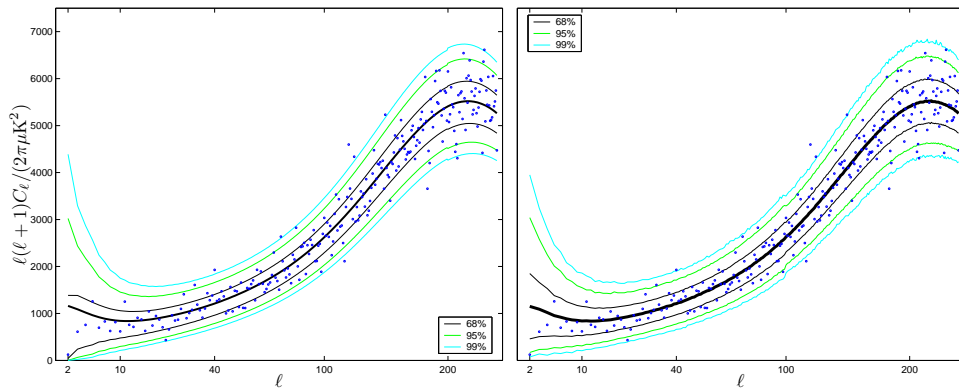


Figure 1. The WMAP power spectrum at low ℓ (where it is signal dominated). The thick line shows a theoretical spectrum (the mean of each value over many realizations). The points are the WMAP \hat{C}_ℓ estimates. The other lines on the left plot are isoprobability contours enclosing 68%, 95% and 99% for the appropriate χ^2 distribution (no noise), lines on the right plot are from pseudo- C_ℓ simulations with noise, where 32%, 5% and 1% of \hat{C}_ℓ realizations are equi-distributed below and above the different lines. This assumes the plotted theoretical model is correct.

In Fig. 1 I have plotted the WMAP temperature \hat{C}_ℓ estimates at $\ell < 250$ where the uncertainty from the noise is not too large. On the full sky and with no noise the maximum likelihood estimators are given by $\hat{C}_\ell = \sum_m |a_{lm}|^2 / (2\ell + 1)$ where the a_{lm} are the coefficients of the spherical harmonic expansion of the CMB temperature on the sky. With cuts around foregrounds like the galaxy and in the presence of noise things are not quite so simple. The values plotted are pseudo- C_ℓ estimators,

^a<http://cosmologist.info/cosmomc/>

which are suboptimal but simple to compute as described in Refs. 9, 1. For a given theoretical model the probability distribution for the \hat{C}_ℓ estimators in any particular realization of the sky is something like a χ^2 with $(2l+1)f_{\text{sky}}^2$ degrees of freedom⁷ (where f_{sky} is the fraction of sky observed). The most likely values of the estimators are therefore actually *below* the mean value (which is the variance of the a_{lm} s predicted by the theoretical model).

The limits in Fig. 1 are approximate and depend on the theoretical model. However there still appear to be a number number of points significantly outside the 99%-confidence lines. There were only three models in 16000 simulations with \hat{C}_{181} lower than the observed value. Outlying points also seem to come in clusters, with the binned power spectrum having some wiggles¹.

2.1. Goodness of fit

The goodness of fit of the observed points to the theoretical model can be assessed by using some kind of effective χ_{eff}^2 value. Ref. 2 reports an effective $\chi_{\text{eff}}^2 = 1431$ based on the likelihood of the theoretical best fit model given the observed \hat{C}_ℓ estimators (including the T-E correlation power spectrum). This may be surprisingly high for the 1342 degrees of freedom, indicating that the standard best fit model has rather low probability given the data. However we note a few technical points:

- The otherwise excellent third-order likelihood approximation used by WMAP⁷ cannot be expected to be accurate for points far from the theoretical value. The values of the likelihoods for the outlier points are probably underestimated and hence giving an artificially high χ_{eff}^2 (and could be skewing parameter estimates, c.f. the section on the quadrupole below.)
- Noise contributions to the variance of the \hat{C}_ℓ have been neglected at $\ell < 100$ even though the noise at $\ell \sim < 100$ is about five times larger than for $\ell \sim > 100$ because fewer frequencies are being used. This overestimates the effective χ_{eff}^2 by about 16 (though has no significant effect on parameter estimates).

The best way to asses whether an effective χ_{eff}^2 is acceptable is by simulations^b. Taking the temperature data only at $\ell < 250$ the observed χ_{eff}^2 is 275, compared to simulations which give $248 \pm 25(1\sigma)$. Thus on these scales where the noise is small there is not strong evidence that anything is amiss. More detailed simulations would be needed on smaller scales to assess whether the $\chi_{\text{eff}}^2 = 963$ over $\ell < 900$ is acceptable or not. See also the section below on the temperature-polarization cross-correlation.

^bMy simulations are rather crude, using single maps and modelling the noise as an isotropic contribution to the variance, which may give wider dispersion than a full simulation. I take the best-fit theoretical model as fixed. My χ_{eff}^2 however accounts for the noise at $\ell < 100$.

2.2. Low quadrupole

The low value of the quadrupole has attracted considerable attention. Although the likelihood of the observed \hat{C}_2 is not that low for standard models (just outside the the 68% likelihood contour in Fig. 1), it is atypical in being low. This means that any model which predicts a low value for the quadrupole will have a higher posterior probability than standard models, because in the new model the low value can be much more typical (and so have much higher likelihood). Some points about the quadrupole value:

- The very low value reported by WMAP of $\hat{C}_2 = 123\mu\text{K}^2$ has unmodelled foreground uncertainties. For example treating the foregrounds differently by using the map from Ref. 10 one obtains a pseudo- C_ℓ estimate $\hat{C}_2 = 184\mu\text{K}^2$. (See also Ref. 11.)
- Ref. 10 claim that the value is low partly because of a coincidence that the galaxy obscures regions of high power. This appears to be a much smaller effect than the foreground uncertainty, as cutting the galactic region from the map used in that paper only lowers the \hat{C}_2 by $\sim 10\%$.
- The exact value of the estimator and the likelihood value for different models is sensitive to how accurately you model the distribution. There are significant differences between a near-optimal analysis using orthogonalized harmonics^{12,13} and the WMAP pseudo- C_ℓ and likelihood parameterization.^c

Without including Lyman- α data on very small scales, some weak evidence for running of the spectral index arises from the low value of the WMAP quadrupole⁶ favouring low theoretical quadrupole values. Other models which favour low quadrupoles are also favoured, though very low values cannot be obtained by only changing the initial power spectrum. This is because about half of the power in the quadrupole comes from the Integrated Sachs Wolfe (ISW) effect, which is sourced by changing potentials along the line of site. Though the large scale power can be decreased to lower the contribution to the quadrupole from last scattering, the ISW contribution cannot be lowered without decreasing the power on smaller scales which is inconsistent with the values of the other C_ℓ . The ISW effect can be decreased by changing the dark energy model from a cosmological constant¹⁴, but when combined with a strange initial power spectrum this is becoming a seriously contrived model.

3. Temperature-polarization correlation

The power spectrum for the correlation between the temperature and E-polarization appears to indicate a significant optical depth to last scattering¹⁵. The relevant

^cSee the plot at <http://cosmologist.info/Sorrento03>. Note that problems of modelling the quadrupole likelihood do not appear to significantly bias parameter constraints for different initial power spectrum models. Even though absolute likelihood values may differ over the range of models for different methods, the models always predict much higher values than the observed value and the slope of the likelihood function is approximately correct over the relevant range.

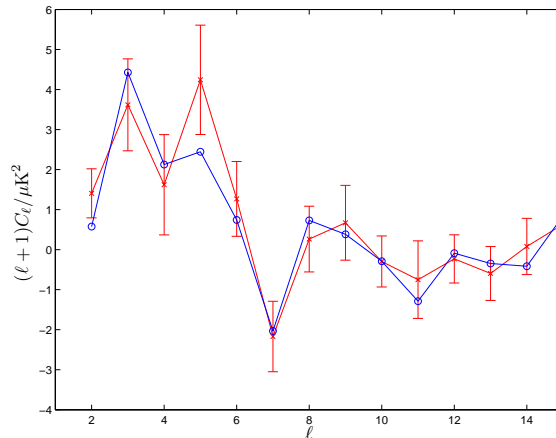


Figure 2. The temperature-polarization correlation power spectrum estimators at $\ell < 16$ provided by WMAP. Error bars are noise only, and both sets of points are available from the WMAP web page.

large scale power spectrum is plotted in Fig. 2, showing the excess of power on large scales that is a signature of reionization. The points with the error bars are those plotted in Ref. 15, whilst the other set of points are the values actually used to compute the likelihoods^{d7}. Presumably these are different analyses of the data, and the significant difference between the points indicate a significant unaccounted for systematic error in the measurement of the TE power spectrum. It is interesting to note that $\ell = 7$ is quite discrepant from theoretical expectations, but does not change significantly between the two analyses. The more than factor of two systematic difference in the TE quadrupole values likely explains the apparent inconsistency with the temperature quadrupole value discussed in Ref. 16. The WMAP likelihood analysis also neglects the correlation between the temperature and TE power spectra, though this does not have a large effect on joint-constraint cosmological parameter values.^e

4. Asymmetries

One of the main theoretical assumptions of most cosmological models is that the universe is statistically isotropic: after removing the local dipole the CMB anisotropy should not have any alignment other than random variations expected from throw-

^d<http://lambda.gsfc.nasa.gov/>

^eIncluding the correlation is no harder than ignoring it at Gaussian order, though it is not trivial to find a third-order likelihood parameterization accounting for the correlations. One can compute the likelihood distributions essentially exactly on large scales using orthogonalized harmonics¹³ (or Monte-Carlo methods¹⁷) with Gaussianity assumptions, which should probably at least be used as a check on any parameterization that is used when better (and more foreground-free) data is available.

ing down realizations of statistically isotropic random fields on the sky. Ref. 18 find in fact that the quadrupole and octopole have some rather unlikely alignment properties, with the octopole being nearly planar, and the axes of the two being closely aligned (however a posteriori statistics for small samples of numbers should be interpreted with a little care).

Interestingly Ref. 19 report that there are also highly significant power anisotropies on larger scales. As an independent minor variation on this theme I computed a single binned \hat{C}_ℓ with $\ell \leq 30$ over half the sky as a function of the axis of the hemisphere, trying 48 different orientations. There is an apparent asymmetry with a hemisphere centred close to the north ecliptic pole giving about 35% less power than the opposite hemisphere. The significance of this result is readily checked by simulating skies according to the predictions of some Gaussian Λ CDM model, and computing the maximum value of the power ratio in the different realizations. The observed value lies 2–3 sigma away from the maximum value of the power ratio, indicating that such a large power deficit over the northern ecliptic hemisphere is rather unlikely. The axis of asymmetry can also be seen (though with less statistical significance) for smaller bins in ℓ , though does not persist to small scales. It appears that there is a region of startlingly low large scale power somewhere around the north ecliptic pole. Colour figures are available on the web^f.

5. Conclusions

It would appear that the WMAP data might be inconsistent with simple isotropic Gaussian cosmological models. Given the past experience (e.g. with BOOMERANG) one's first suspicion naturally falls on the data, and the alignment of power with the ecliptic (the axis of symmetry of the observation) may be a hint in this direction. However the features appear to be quite robust, and deficits in power are quite hard to explain with foregrounds. One possible explanation for the outlying points in the power spectrum might be non-Gaussianity, with a_{lm} s being more likely close to zero or large than if they had a Gaussian distribution.

Whatever the resolution of the puzzle, the current determination of cosmological parameters assuming everything fits our assumptions is potentially misleading. A small number of outlier points with large weight under the Gaussianity assumption could be skewing parameter estimates if they are included in the analysis but in fact have their origin in some other systematic or non-Gaussian physics. However the concordance of parameter estimates does suggest that this effect may be small. In any case it should be a matter of some priority to check the WMAP results.

Ref. 17 has presented a very nice Monte Carlo method for computing cosmological parameter likelihoods essentially exactly if the CMB really is Gaussian and isotropic. This can include foreground uncertainties consistently, and avoids all the problems with computing accurate likelihood values from a set of \hat{C}_ℓ estimators.

^f<http://cosmologist.info/Sorrento03>

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References

1. G. Hinshaw et al. First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Angular power spectrum. *Astrophys. J. Suppl.*, 148:135, 2003, astro-ph/0302217.
2. D. N. Spergel et al. First year wilkinson microwave anisotropy probe (WMAP) observations: Determination of cosmological parameters. *Astrophys. J. Suppl.*, 148:175, 2003, astro-ph/0302209.
3. Carlo R. Contaldi, Henk Hoekstra, and Antony Lewis. Joint CMB and weak lensing analysis: Constraints on cosmological parameters. *Phys. Rev. Lett.*, 90:221303, 2003, astro-ph/0302435.
4. Uros Seljak, Patrick McDonald, and Alexey Makarov. Cosmological constraints from the CMB and Ly-alpha forest revisited. *Mon. Not. Roy. Astron. Soc.*, 342:L79, 2003, astro-ph/0302571.
5. Will J. Percival et al. The 2dF Galaxy Redshift Survey: The power spectrum and the matter content of the universe. *MNRAS*, 327:1297, 2001, astro-ph/0105252.
6. S. L. Bridle, A. M. Lewis, J. Weller, and G. Efstathiou. Reconstructing the primordial power spectrum. *MNRAS*, 342:L72, 2003, astro-ph/0302306.
7. L. Verde et al. First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Parameter estimation methodology. *Astrophys. J. Suppl.*, 148:195, 2003, astro-ph/0302218.
8. Antony Lewis and Sarah Bridle. Cosmological parameters from CMB and other data: a Monte- Carlo approach. *Phys. Rev.*, D66:103511, 2002, astro-ph/0205436.
9. B. D. Wandelt, E. Hivon, and K. M. Górski. Cosmic microwave background anisotropy power spectrum statistics for high precision cosmology. *Phys. Rev.*, D64:083003, 2001, astro-ph/0008111.
10. Max Tegmark, Angelica de Oliveira-Costa, and Andrew Hamilton. A high resolution foreground cleaned CMB map from WMAP. 2003, astro-ph/0302496.
11. G. Efstathiou. Myths and truths concerning estimation of power spectra. 2003, astro-ph/0307515.
12. K. M. Górski. On determining the spectrum of primordial inhomogeneity from the COBE DMR sky maps: I. method. *Astrophys. J. Lett.*, 430:85, 1994, astro-ph/9403066.
13. D. J. Mortlock, A. D. Challinor, and M. P. Hobson. Orthogonalisation of scalar basis functions on an incomplete sky. *MNRAS*, 330:405, 2002, astro-ph/0008083.
14. J. Weller and A. M. Lewis. Large scale cosmic microwave background anisotropies and dark energy. 2003, astro-ph/0307104.
15. A. Kogut et al. Wilkinson Microwave Anisotropy Probe (WMAP) first year observations: TE polarization. *Astrophys. J. Suppl.*, 148:161, 2003, astro-ph/0302213.

16. Olivier Dore, Gilbert P. Holder, and Abraham Loeb. The CMB quadrupole in a polarized light. 2003, astro-ph/0309281.
17. Benjamin D. Wandelt, David L. Larson, and Arun Lakshminarayanan. Global, exact cosmic microwave background data analysis using Gibbs sampling. 2003, astro-ph/0310080.
18. Angelica de Oliveira-Costa, Max Tegmark, Matias Zaldarriaga, and Andrew Hamilton. The significance of the largest scale CMB fluctuations in WMAP. 2003, astro-ph/0307282.
19. H. K. Eriksen, F. K. Hansen, A. J. Banday, K. M. Gorski, and P. B. Lilje. Asymmetries in the CMB anisotropy field. 2003, astro-ph/0307507.